# Pricing Game of Celebrities in Sponsored Online Viral Marketing



### Cascade with Varying Activation Probability model

ICM

For a node u, after getting active, it has a single chance to activate each of its inactive son v, success with probability p

Activation probability p varies

 $p(s) = \begin{cases} \frac{p_m}{1 + ce^{-r_1 s}}, & s \le tp\\ -r_2(s - tp) + p', & otherwise \end{cases}$ 

**CVAP** model

Influence function  $\sigma$  is submodular '





**Celebrity Selection** Greedy algorithm provides  $1 - \frac{1}{2}$  optimality

# Pricing Game of Celebrities

С

x

y

 $\boldsymbol{V}$ 

=3

 $\sigma(x)$ 

 $\alpha_{xv}$ 

 $\delta_{v}$ 

 $\sigma(v)$ 

#### The settings:

- Graph G = (C, V, E)
- $\sigma$  is provided as an oracle
- Information complete
- No cost, no budget limit

#### The focus:

$$NASH = \begin{cases} \boldsymbol{p} \in \mathbb{R}_{+}^{|C|} \mid u_i(p_i, \boldsymbol{p}_{-i}) \ge u_i(p'_i, \boldsymbol{p}_{-i}), \\ \forall p'_i \in \mathbb{R}_{+} \end{cases}$$

The simple greedy algorithm	The double greedy algorithm	$\delta_i + \sigma(c_i S_{i-1})$
Theorem 1. $NASH = \{\delta\}$ , if $ C  = 2$	Theorem 4. $NASH = \{ \boldsymbol{\varphi} \}$	$\varphi_{i} = \frac{\varphi_{i}}{2}$ $S_{i} = \{c_{1}, c_{2}, \dots, c_{i}\}$
Theorem 2. $ NASH  = 1$ , if $ C  = 3$		
Theorem 3. Those cannot be guaranteed, if $ C  \ge 4$ : • NASH $\neq \emptyset$ (existence) • NASH = $\emptyset$ (non-existence) • If NASH $\neq \emptyset$ , [NASH] = 1 (uniqueness)	Theorem 5. $u(X_2(\boldsymbol{\varphi})) \in \left[\frac{1}{2}u(S^*), \frac{1}{2}\sigma(C)\right]$	$ = \sum_{S^*} X_2 \colon \mathbb{R}^{ C }_+ \to 2^C $ $S^* = \max_{S \subseteq C} u(S) $ $Zhiyi LU - zhiyi V@gmail.com$